# A Mixed Surface Volume Integral Formulation for the Modeling of High Frequency Coreless Inductors

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**An original integral formulation dedicated to the high frequency modeling of electromagnetic systems without magnetic materials is presented. The total current density (*i.e.* conduction plus displacement currents) is approached by facet elements so that resistive, inductive and capacitive effects are all modeled. The method avoids moreover the volumic mesh of the conductors, which is too dense at high frequencies, due to the skin and proximity effects appearing *e.g.* in wound inductors. Surface impedance boundary conditions are employed to that end. The formulation is general and suitable for non simply connected domains. It is experimentally validated on a coreless wound inductor, using an impedance analyzer.**

*Index Terms*—**Computational electromagnetics, Integral equations, Electromagnetic compatibility**

## I. Introduction

Current progresses in power electronics are enabling the use of higher switching frequencies, up to several MHz [1]. This permits to reduce the volume and the mass of the passive components, which is critical for transportation applications. This frequency range is also used for the design of wireless power transfer systems [2] (wireless charging, transcutaneous energy transmitters, etc.). At such frequencies, parasitic capacitive effects cannot be neglected, as they may induce ElectroMagnetic Compatibility (EMC) problems, or influence the resonant frequency of a wireless power transmitter.

The Partial Element Equivalent Circuit (PEEC) method offers a framework for the modeling of such physical phenomena [3]. It is an integral method which associates an RLC circuit to a meshed geometry, and which does not require the mesh of the air. An adaptation to unstructured meshes, based on the use of facet elements, has recently been proposed [4], in magnetodynamics. However, the skin and proximity effects, which appear in the conductors at such frequencies, lead to very fine meshes. Therefore, in this paper, the formulation of [4] is extended so as to avoid the volumic mesh of the conductors. Surface Impedance Boundary Conditions (SIBCs) are employed to that end. The capacitive effects are moreover taken into account using the formalism presented in [5]. The method is described in Section II, and is experimentally validated in Section III on a coreless wound inductor.

## II. Formulation

The magnetic vector and electric scalar potentials, $\mathbf{A}$ and $V$ respectively, are first expressed in their integral form:

$$\mathbf{A}(\mathbf{x}) = \frac{\mu_0}{4\pi}\left(\int_{\Omega_J}\frac{\mathbf{J}}{r}\mathrm{d}\Omega + \int_{\Omega_D} j\omega\frac{\mathbf{P}}{r}\mathrm{d}\Omega\right), \quad (1)$$

$$\frac{\partial V}{\partial t}(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\left(\int_{\Omega_J}\mathbf{J}\,\mathrm{grad}\frac{1}{r}\mathrm{d}\Omega + \int_{\Omega_D} j\omega\mathbf{P}\,\mathrm{grad}\frac{1}{r}\mathrm{d}\Omega\right), \quad (2)$$

in the frequency domain, with $j$ the imaginary number, $\omega$ the angular frequency, $\mathbf{J}$ the conduction current density, $\mathbf{P}$ the electric polarization, $\mu_0$ ($\epsilon_0$) the magnetic permeability (electric permittivity) of vacuum, $r$ the distance between the field sources and point $\mathbf{x}$. The integrals are defined on $\Omega_J$ ($\Omega_D$), namely the domain containing the conductors (the dielectrics). The potentials are injected into the electric field $\mathbf{E}$ equation:

$$\mathbf{E} + j\omega\mathbf{A} + \mathrm{grad}V = 0 \quad (3)$$

A Galerkin procedure is then applied on eq. (3), by using the total current density $\mathbf{J}_t$ as the problem unknown :

$$\mathbf{J}_t = \mathbf{J} + j\omega\mathbf{D} = \mathbf{J} + \epsilon j\omega\mathbf{E} = (\sigma + j\omega\epsilon)\mathbf{E}, \quad (4)$$

with $\mathbf{D}$ the electric flux density and $\sigma$ the electric conductivity. First order volume facet elements defined on $\Omega = \Omega_J \cup \Omega_D$ are employed [4], [5], which permits to translate the field equations into circuit equations supported by the dual mesh. This leads to the resolution of the following system of equations:

$$\left([R] + j\omega[L] + \frac{1}{j\omega}[P]\right)\{\mathbf{I}\} = \{\Delta V\}, \quad (5)$$

with $\{\mathbf{I}\}$ the total branch currents crossing the facets, $\{\Delta V\}$ the branch voltages, $[R]$ a sparse finite element resistance matrix, $[L]$ and $[P]$ the full inductance and capacitance matrices.

However, at high frequencies, the skin and proximity effects in the massive conductors lead to very fine meshes of $\Omega_J$. Therefore, in this paper, the formulation (5) is extended so as to avoid the volumic mesh of the conductors. $\mathbf{J}_t$ is approached by assuming that $\sigma = 0$ in $\Omega_D$ and $\sigma >> \omega\epsilon$ in $\Omega_J$:

$$\mathbf{J}_t = \underbrace{\mathbf{J}_t|_{\Omega_D}}_{=j\omega\mathbf{D}} + \underbrace{\mathbf{J}_t|_{\Gamma_J}}_{=\mathbf{J}} = \sum_{k\in\Omega_D}\mathbf{w}_k I_k + \sum_{i\in\Gamma_J}\mathbf{w}_{S,i} I_{S,i} \quad (6)$$

with $\Gamma_J$ the boundary between $\Omega_J$ and $\Omega_D$ (or air), $\mathbf{w}_k$ the facet basis functions (in $[\mathrm{m}^{-2}]$), $\mathbf{w}_{S,i}$ the surface facet basis functions (in $[\mathrm{m}^{-1}]$). $I_k$ is the total current crossing a facet $k$ of $\Omega_D$ (in [A]), whereas $I_{S,i}$ is the total current crossing an edge $\Gamma_{J,i}$ of $\Gamma_J$, in $[\mathrm{A\,m}^{-1}]$ (see Fig.1(a)).

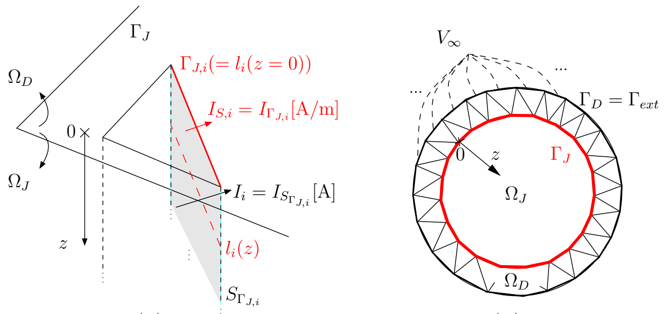


Fig. 1. (a) focus on an element of $\Gamma_J$ and (b) example of discretization of a conductor with its surrounding insulation.

The total current $I_i$ in the skin layer (*i.e.* crossing the surface $S_{\Gamma_{J,i}}$ in $\Omega_J$) needs to be expressed as a function of the edge currents $I_{S,i}$. To that end, a first order SIBC is employed [6]:

$$I_i = \int_0^{\infty} I_{l_i(z)} dz = \int_0^{\infty} I_{S,i} e^{-\frac{1+j}{\delta}z} dz = \frac{\delta}{1+j} I_{S,i}, \tag{7}$$

with $\delta$ the skin depth and $z$ the distance inside the conductor. The total current density $\mathbf{J}_t$ (6) becomes therefore:

$$\mathbf{J}_t = \mathbf{J}_t|_{\Omega_D} + \mathbf{J}|_{\Gamma_J} = \sum_{k\in\Omega_D} \mathbf{w}_k I_k + \sum_{i\in\Gamma_J} \mathbf{w}_{S,i} \frac{1+j}{\delta} I_i \tag{8}$$

In this summary, we focus on the surface term only (*i.e.* the second sum of equation (8)), and on the corresponding modifications in the matrices of (5). To that end, equation (3) is projected on $\Gamma_J$ with the $\mathbf{w}_{S,i}$:

$$\int_{\Gamma_J} \mathbf{w}_{S,i} \frac{\mathbf{J}}{\sigma} d\Gamma + j\omega \frac{\mu_0}{4\pi} \int_{\Gamma_J} \mathbf{w}_{S,i} \left( \int_{\Omega_J} \frac{\mathbf{J}}{r} d\Omega \right) d\Gamma$$
$$+ j\omega \frac{\mu_0}{4\pi} \int_{\Gamma_J} \mathbf{w}_{S,i} \left( \int_{\Omega_D} \frac{\epsilon - \epsilon_0}{\epsilon} \frac{\mathbf{J}_t}{r} d\Omega \right) d\Gamma + \int_{\Gamma_J} \mathbf{w}_{S,i} \mathrm{grad} V d\Gamma = 0 \tag{9}$$

It can be shown that the last term in equation (9) corresponds to the branch voltages $\Delta V_i$ between elements of $\Gamma_J$. The volume integral on $\Omega_J$ is moreover approached by:

$$\int_{\Omega_J} \frac{\mathbf{J}}{r} d\Omega = \int_{\Gamma_J} \int_0^{\infty} \frac{\mathbf{J}(z=0)}{r} e^{-\frac{1+j}{\delta}z} dz d\Gamma \simeq \frac{\delta}{1+j} \int_{\Gamma_J} \frac{\mathbf{J}}{r} d\Gamma \tag{10}$$

By combining equations (8), (9) and (10), the matrices $[R]$ and $[L]$ of formulation (5) are therefore augmented with the following pure surface terms ($i, j \in \Gamma_J$):

$$R_{ij} = \int_{\Gamma_J} \frac{\mathbf{w}_{S,i} \mathbf{w}_{S,j}}{\sigma^*} d\Gamma \quad L_{ij} = \frac{\mu_0}{4\pi} \int_{\Gamma_J} \mathbf{w}_{S,i} \left( \int_{\Gamma_J} \frac{\mathbf{w}_{S,j}}{r} d\Gamma \right) d\Gamma \tag{11}$$

with $\sigma^* = \sigma\delta/(1+j)$ the equivalent conductivity. If $i \in \Gamma_J$ and $j \in \Omega_D$, the surface-volume coupling term reads:

$$L_{ij} = \frac{\mu_0}{4\pi} \int_{\Gamma_J} \mathbf{w}_{S,i} \left( \int_{\Omega_D} \frac{\epsilon - \epsilon_0}{\epsilon} \frac{\mathbf{w}_j}{r} d\Omega \right) d\Gamma \tag{12}$$

The matrix $[P]$, which is defined by adding capacitive branches between the facets belonging to the external border $\Gamma_{ext}$ and an external reference node $V_\infty$ (see Fig.1(b)), remains unchanged, compared to [5].

## III. Test case and discussion

The formulation is tested on a 19-turn, 2 layer coreless wound inductor, with conductors of radius 0.1 mm surrounded by an insulating layer of 0.0215 mm, with $\epsilon_r = 2.5$. The coil-former is characterized by an $\epsilon_r = 3$. A circuit solver has been employed to solve the system (5), leading to 18735 unknowns (or independent circuit loops). This has to be compared to the 52000 unknowns of the full volumic formulation [5], which shows the interest of the proposed method.

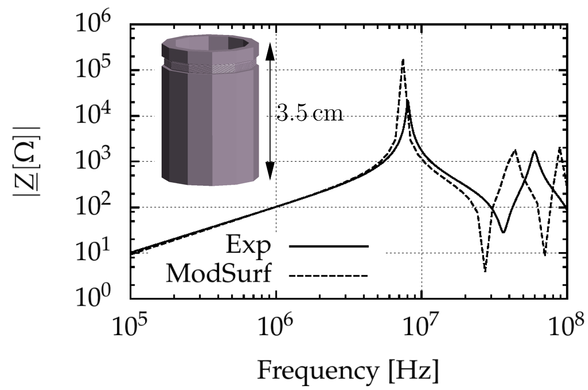


Fig. 2. Impedance of the test coil, measurement and model.

An impedance analyzer Agilent 4294A has been used to measure the impedance of the component. Figure 2 depicts the module of the impedance of the test coil, in the case of the measurement (plain lines) and the model (dotted lines). One can see that the position of the first resonance is quite similar for the two curves, which is less valid for the higher order resonances. This can be associated to uncertainties regarding the parameters of the test coil (geometry as well as material parameters). A validation on a full-wave finite element solver will be proposed in the full paper.

Moreover, the amplitude of the peaks appears to be higher with the model, which may be due to an inaccurate estimation of the coil resistance with the first order SIBC. Higher order SIBCs will be tested in the full paper.

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